HW1 Solution

Monday, November 23, 2009
11:33 AM

1

(a) $x(t) = A_c m(t) \xrightarrow{\mathcal{F}} X(f) = A_c M(f)$

So, $X(f)$ is also bandlimited to W.

$u(t) = x(t) + \sqrt{2}\cos(\omega_c t)$ $\qquad \omega_c = 2\pi f_c$

$v(t) = u^2(t) = \left(x(t) + \sqrt{2}\cos(\omega_c t)\right)^2$

$= x^2(t) + 2\sqrt{2}\,x(t)\cos(\omega_c t) + \underbrace{2\cos^2(\omega_c t)}_{1+\cos(2\omega_c t)}$

$= \underbrace{\left(1 + x^2(t)\right)}_{\text{BPF} \to 0} + 2\sqrt{2}\,x(t)\cos\omega_c t + \underbrace{\cos(2\omega_c t)}_{\text{BPF} \to 0}$

Note 1: $x^2(t) \xrightarrow{\mathcal{F}} X(f) * X(f)$

So, $x^2(t)$ is bandlimited to $2W$

Because $f_c \gg W$, the spectrum of $x^2(t)$ will not be in the passband of the BPF which centers around f_c.

Note 2: The term $\cos(2\omega_c t)$ is at frequency $2 \times f_c$ which again is outside the passband.

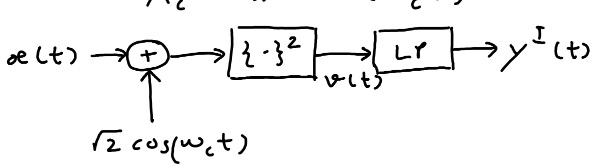
$y(t) = \text{BPF}\{v(t)\}$

$= 2\sqrt{2}\,x(t)\cos\omega_c t$

$= \boxed{2\sqrt{2}\,A_c\, m(t)\cos\omega_c t}$

(b) Assume

$x(t) = A_c m(t)\sqrt{2}\cos(\omega_c t)$



From the above figure,

$v(t) = \left(x(t) + \sqrt{2}\cos(\omega_c t)\right)^2$

$= 2\cos^2(\omega_c t)\left(A_c m(t) + 1\right)^2$

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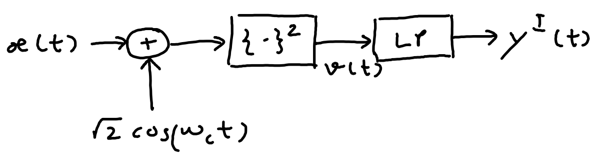


From the above figure,

$v(t) = \left(x(t) + \sqrt{2}\cos(\omega_c t)\right)^2$

$= 2\cos^2(\omega_c t)\left(A_c m(t) + 1\right)^2$

$= 1 + \cos(2\omega_c t)\left(\underbrace{A_c^2 m^2(t)}_{\substack{\text{spectrum is from} \\ [-2W,\, 2W]}} + 1 + 2\underbrace{A_c m(t)}_{\substack{\text{spectrum is from} \\ [-W,\, W]}}\right)$

$g(t)$

$$= g(t) + \underbrace{g(t)\cos(2\omega_c t)}_{\xrightarrow{LPF} 0}$$

Note 1: We know that $g(t)$ is band limited to $[-2w, 2w]$ because all of its terms are band limited to $[-2w, 2w]$. So, only some parts of it will pass through the LPF.

Note 2: $g(t)\cos(2\omega_c t)$ is centered @ $2f_c$ and therefore will not pass thought the LPF.

$$y^I(t) = LPF\{v(t)\}$$
$$= LPF\{g(t)\}$$
$$= \boxed{1 + 2A_c m(t) + LPF\{A_c^2 m^2(t)\}}$$

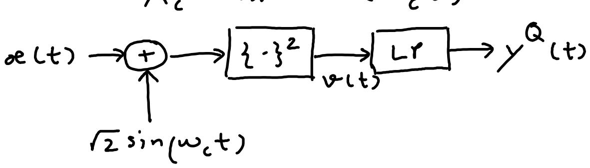
This term has spectrum beyond IW So, only a portion of it will pass through the LPF.

$y^I(t)$ is $\boxed{\text{not}}$ proportional to $m(t)$.

Hence, this block diagram does not work as a demodulator.

(c) Assume

$x(t) = A_c m(t)\sqrt{2}\cos(\omega_c t)$ as in part (b).



We then have

$$v(t) = \left(x(t) + \sqrt{2}\sin(\omega_c t)\right)^2$$
$$= 2\left(A_c m(t)\cos(\omega_c t) + \sin(\omega_c t)\right)^2$$
$$= 2\left(A_c^2 m^2(t)\cos^2(\omega_c t) + A_c m(t)\cos(\omega_c t)\sin(\omega_c t) + \sin^2(\omega_c t)\right)$$
$$= 2\left(A_c^2 m^2(t)\cos^2(\omega_c t) + \sin^2(\omega_c t)\right) + A_c m(t)\sin(2\omega_c t)$$
$$= 2\left(\left(A_c^2 m^2(t) - 1\right)\cos^2(\omega_c t) + 1\right) + A_c m(t)\sin(2\omega_c t)$$
$$= 2 + \left(A_c^2 m^2(t) - 1\right)\left(1 + \underbrace{\cos(2\omega_c t)}_{\xrightarrow{LPF} 0}\right) + \underbrace{A_c m(t)\sin(2\omega_c t)}_{\xrightarrow{LPF} 0}$$

$$y^Q(t) = 2 + LPF\{A_c^2 m^2(t)\} - 1$$
$$= \boxed{LPF\{A_c^2 m^2(t)\} + 1}$$

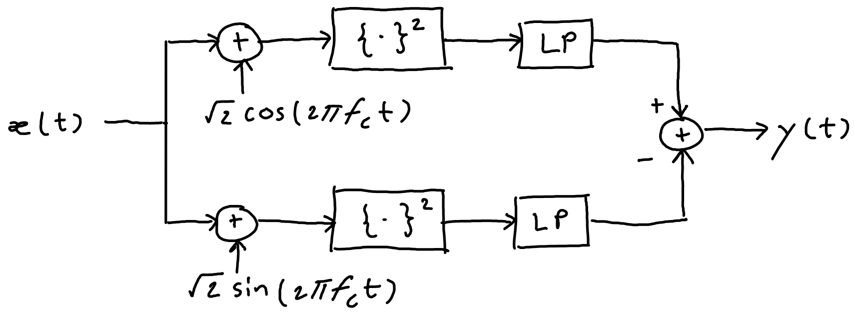
$$= \boxed{LPF\left\{A_c^2 m^2(t)\right\} + 1}$$

(d) Observe that

$y^I(t) - y^Q(t) = 2A_c m(t)$ which is the desired output of a successful DSB-SC demodulator.

$y^I(t)$ from (b), $y^Q(t)$ from (c)

Hence, the following block diagram would work:



2

(a) $y(t) = \left(m(t) + \sqrt{2}\cos(2\pi f_0 t)\right)^3$

$$= m^3(t) + 3m^2(t)\sqrt{2}\cos\omega_0 t + \underbrace{3m(t)\,2\cos^2\omega_0 t}_{} + \underbrace{(\sqrt{2})^3\cos^3(\omega_0 t)}_{}$$

$= 3m(t)\left(1+\cos 2\omega_0 t\right)$

$= 3m(t) + 3m(t)\cos(2\omega_0 t)$

$= \frac{3}{\sqrt{2}}\cos(\omega_0 t) + \frac{1}{\sqrt{2}}\cos(3\omega_0 t)$

$2\cos^2(\theta) = 1 + \cos(2\theta)$

$2\cos^3(\theta) = \cos\theta + \cos\theta\cos 2\theta$

$= \cos\theta + \frac{1}{2}\cos\theta + \frac{1}{2}\cos 3\theta$

$= \frac{3}{2}\cos\theta + \frac{1}{2}\cos 3\theta$

We want $z(t) = m(t)\sqrt{2}\cos(\omega_c t)$.

We see that the only term in $y(t)$ that has the form

constant $\times$ m $\times$ $\cos(\;)$

is $3m(t)\cos(2\omega_0 t)$.

Therefore, we will center the passband to cover this part and adjust the gain to make the output the same as $z(t)$.

In particular,

We need to make $\boxed{2f_0 = f_c.}$ So, $\boxed{f_0 = f_c/2.}$

Let $H_{BP}(f) = \begin{cases} c, & |f - f_c| \le W \\ c, & |f + f_c| \le W \\ 0, & \text{otherwise} \end{cases}$

Then,

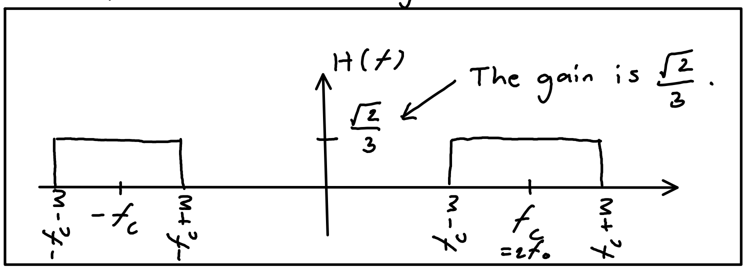
$$z(t) = \underbrace{C \times 3}\, m(t) \cos(2\omega_0 t)$$

$\downarrow$

we need $C \times 3 = \sqrt{2}$

$$C = \frac{\sqrt{2}}{3}$$

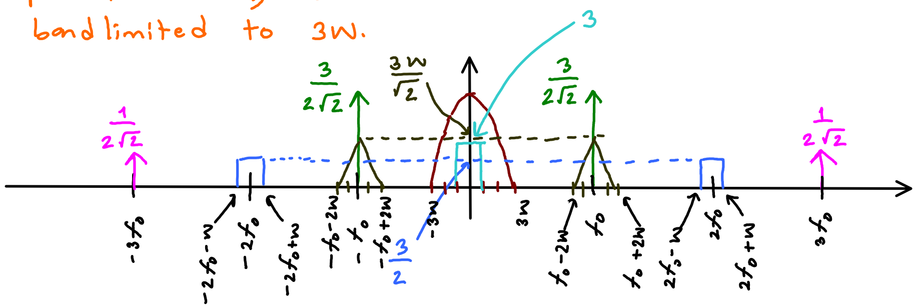
The plot of $H(f)$ is given below:



(b) From (a), we have

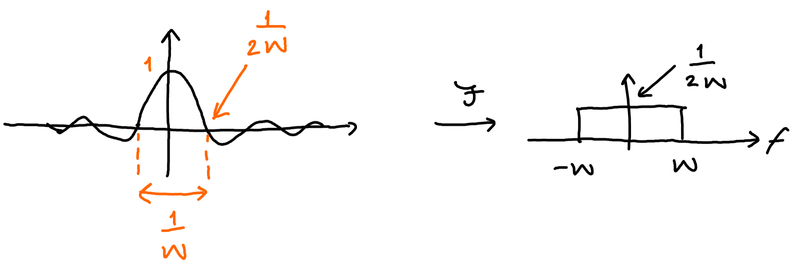
$$y(t) = m^3(t) + 3\sqrt{2}\, m^2(t)\cos(\omega_0 t) + 3m(t)\cos(2\omega_0 t) + \frac{1}{\sqrt{2}}\cos(3\omega_0 t)$$
$$+ 3m(t) \qquad + \frac{3}{\sqrt{2}}\cos(\omega_0 t)$$

Without trying to make and accurate plot for $m^3(t)$, we know that it is band limited to $3W$.

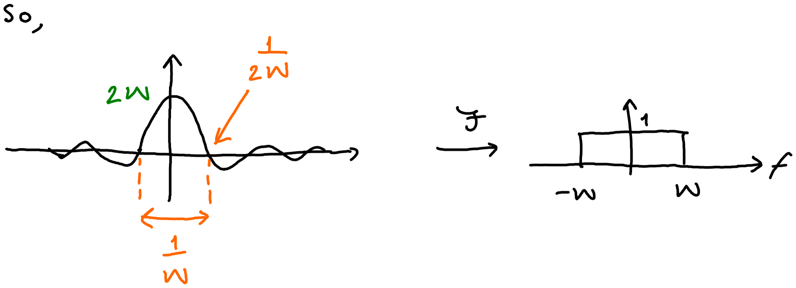


(c) $z(t) = m(t)\sqrt{2}\cos(2\pi f_c t)$

We know that



So,



$z(t)$ is the above sinc. function multiplied

$Z(t)$ is the above sinc. function multiplied by $\sqrt{2}\cos(2\pi f_c t)$.

Because $f_c \gg W$, we know that

$$\frac{1}{W} \gg \frac{1}{f_c}$$

$\uparrow$ period of cos.

So, the sinc function becomes the envelope of the cosine carrier.